Reconstruction of Images with Poisson Noise

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Abstract

This paper addresses quantitatively the problem of influence of statistical uncertainties embedded in the recorded image on uncertainties of the reconstructed image. In the analysis we use iterative maximum likelihood algorithm *ANDRIL* (described by Sylwester and Sylwester 1998) developed for massive deconvolution of flare images obtained by the Soft X-ray Telescope (SXT) on *Yohkoh.* We illustrate the "ill-conditioned" nature of the image reconstruction problem and suggest ways to reduce, at least partly, propagation of noise to the reconstructed image.

Key words: Sun: X-rays - Techniques: image processing, noise propagation

1 Introduction

In the preceding paper (Sylwester and Sylwester 1998, Paper I) we have described details of the **A**ccelerated **N**oise **D**amped **RI**chardson–**L**ucy algorithm (ANDRIL) used for massive deconvolution of flare X-ray images obtained by the Soft X-ray Telescope (SXT). The instrument is operating at present on board the Japanese *Yohkoh* satellite. Deconvolution is performed in order to remove the image blurring due to the instrument point spread function (PSF) and increase numerically the spatial resolution of the images. The increase of resolution is achieved by allowing for pixel subsampling within the algorithm. It has been noticed in Paper I that using pixel subsampling approach it is possible to increase the angular resolution on the restored (deconvolved) SXT images by factor $2\div3$ bringing it down below 1 arcsec threshold. In the present paper we discuss how the statistical uncertainties present in the data (the Poisson noise) influence the deconvolved image pattern. We illustrate the problem using test synthetic examples and suggest ways how to lessen the propagation of uncertainties in two scenarios: slowly and fast changing source brightness. In the individual synthetic examples considered, the statistical noise is imposed into the input images. Next, the comparisons are performed between the magnitude of root mean square (rms) input fluctuations and corresponding rms fluctuations in the deconvolved image. Derived conclusions are useful in the interpretation of deconvolved images for all SXT filters, including the most important pair of Al12 and Be119, normally used for determination of thermodynamic characteristics of bright fine structures.

2 Deconvolution of Noisy Images

In Paper I our discussion has been focused preferentially on the formal description of the ANDRIL algorithm and these of its properties which are important while restoring the data without the noise (or when the noise was present for the background signal only). In the present contribution we investigate the situation when the input image for deconvolution (assumed for convenience to be of 20×20 SXT pixels size) is subject to the statistical Poisson noise. In Fig. 1 we present example results of deconvolution in case when the test source has been selected to be of ,,V" shape with two symmetric arms. We have decided to decrease the arms' intensities close to the crossing point (see the upper leftmost image) in order to avoid problems with artificial peak present on deconvolved images at the place where linear structures intersect (see discussion in Paper I). In the upper row of Fig. 1 the left image represents the assumed source intensity pattern (the model), the middle is the image as would be observed by the SXT Be119 filter and the right is the ANDRIL deconvolved image obtained using subdivision of each pixel into 5×5 subpixels (if not stated otherwise, this pattern of subsampling has been generally adopted). The rightmost image has been calculated assuming the noise to be absent (for completeness). The middle and bottom rows show the deconvolved images as obtained for cases when the statistical noise is embedded individually (independent perturbation patterns) into the corresponding input image. The maximum ,,observed" signal ($O_{\max}$) has been chosen to be 100 and 1000 (counts) for cases presented in the middle and bottom rows respectively (these selections correspond to input signal to noise ratio of $S/N \sim 10$ and 30).

Figure 1: Example showing the influence of the Poisson data noise on the results of deconvolution. In the upper row the left image shows the assumed source intensity pattern (model), the middle is the image (i.e. the input for deconvolution) as would be observed by SXT (appropriate Be119 filter PSF function has been adopted) and the right image is the result of deconvolution when the noise is absent. The lower two rows present deconvolved images for cases when the input image is noisy. The three images in each row correspond to three independent sets of random noise pattern.

It is seen in the Figure that the intensity distribution in deconvolved images is quite sensitive to the presence of noise. It is especially pronounced (as expected), in case of low statistics (low S/N). One may observe that spurious, localized brightenings arise in the process of deconvolution. This tendency of the algorithm to develop high contrast well localized sources is very much preserved notwithstanding that within the ANDRIL algorithm special ,,damping" of the noise is forced (as described in Paper I). It is seen however, that with increasing of S/N ratio from 10 to 30 (middle and bottom rows respectively) the quality of restored image improves very much and the restored pattern becomes closer to the test model.

Development of spurious maxima in the process of deconvolution is very unpleasant characteristic of ANDRIL indeed. This property should never be forgotten while interpreting results of deconvolution. It is especially troublesome in cases where only a single observed image is available preventing to the large extent possibility of identifying real weak structures. Identification of significant weak point like sources using the maximum likelihood approach has been discussed for case of *ROSAT* X-ray observations by Cruddace, Hasinger and Schmitt, (1988).

It is feasible to look for a possible ways to overcome, at least partly, serious problems caused by Poisson noise. In the next Chapter we discuss in this context two presently investigated exposure averaging approaches and in Chapter 4 we describe semi-empirical method which helps to suppress quite effectively the role of noise in a particular single observed frame.

3 Noise Reduction by Averaging

In cases when the observing cadence of the same field is executed fast relative to time scale of source brightness variations, it seems natural to expect that it would be possible to smooth out the noisy component and come closer to the ,,true" solution by frame averaging. One may consider two approaches to averaging:

- performing multiple deconvolutions for individual observed frames with subsequent averaging of the results (see B in Table 1),
- averaging the input frames with the following deconvolution (see C in Table 1).

We have cross-compared the results ot both approaches by making a number of numerical tests. We considered in more detail the first approach since in this case the results give some insight into the magnitude of uncertainties propagation in deconvolution (see Chapter 5). By taking described ,,V" shaped brightness distribution as typical model shape, we calculated the resulting ,,observed" image and made it a subject to random Poisson noise. Next, the deconvolution has been performed separately for each noisy image. We analysed the results of deconvolution as averaged over 5, 10, 20 and 40 frames.

In Fig. 2 we show the results. In the upper left, the test pattern is presented (as in Fig. 1). In the upper right a particular deconvolved intensity distribution is depicted (no averaging). As already shown in Fig. 1, general shape resembling the letter ,,V" can be recognized on the deconvolved image, but the intensity along the arms is by no means homogeneous (as is the case in

Figure 2: In the upper row the comparison of the test model (left) with a particular deconvolved image obtained from the noisy input with maximum $S/N \sim 10$ (right) is shown. In the lower rows, the left panel represents the average deconvolved image as obtained from 5, 10 and 40 individual deconvolution runs. The images to the right show corresponding maps of the relative standard deviations. The darker shades correspond to larger relative fluctuations in the deconvolved images due to the noise present. In the bottom right image the input observational pixel grid (dotted) is superposed for the comparison.

the test pattern). In the lower rows we present the results of averaging of deconvolved patterns over 5, 10 and 40 frames (left panel) together with corresponding maps (right panel) of the rms fluctuations as determined for each subpixel separately. The comparison of test and averaged patterns clearly indicates that the role of noise decreases as the number of frames used in the averaging grows. Satisfactory match to the test pattern is observed in case of averaging over *several* runs for moderate signal to noise ratio.

In order to quantify the quality ot the reconstruction, we introduce the parameter q (distance between two surfaces representing the model and restored images) as calculated according to the following expression:

$$q = \frac{1}{n_{\mathrm{i}}} \sum_{\mathrm{i}} \left(M_{\mathrm{i}} - \overline{\varphi_{\mathrm{i}}} \right)^2 , \tag{1}$$

where M_{i} stands for the model test brightness in subpixel ,,i" and the summation is performed over all subpixels bright enough in the initial model, i.e. those where number of counts are at least one (n_{i} represents the number of such image elements). Distribution $\overline{\varphi_{\mathrm{i}}}$ represents the average deconvolved intensities in the corresponding pixels. The integral of both M_{i} and $\overline{\varphi_{\mathrm{i}}}$ in Eq. (1) are normalized to 1 in order to allow cross-comparison between cases with different maximum intensities. For distributions having the same shape $q \equiv 0$ and value of q grows as the shapes differ more and more. In the case of deconvolution without noise, this value is $q = 3.8 \times 10^{-10}$.

Table 1: **Deconvolution quality for noisy data**

A: Dependence on intensity

$O_{\max}$	100	500	1000	3000
q (10^{-9})	50±12	6.4±1.4	2.3±0.4	0.73±0.09

B: Deconvolved and averaged, $O_{\max} = 100$

avaraging over:	1	5	10	20	40
q (10^{-9})	50	9.7	6.8	3.6	2.4

C: Averaged and deconvolved, $O_{\max} = 100$

avaraging over:	1	5	10	20	40
q (10^{-9})	50	9.2	5.7	2.8	1.7

In Table 1 the results of image reconstruction are compared for various types of tests described above and in the rest of the paper. From part A of the Table it is seen that the quality of reconstruction improves fast as the intensity in the input image increases. Part B shows to what extent the averaging over a number of deconvolved images improves the quality of reconstruction and in part C we present corresponding results when deconvolution is performed over the input frames averaged.

The results presented in Table 1 indicate that summation of the frames before performing the deconvolution is better strategy since it saves a lot of computer time, and better recovers the model's shape. However, such avaraging is physically sounded in case of slowly changing structures (like non-flaring active regions). It is known that solar flare brightness variations happen to take place on time scales of fraction of second to few seconds. This is usually faster than the cadence executed by SXT, and therefore image averaging is *not* a sounded way to lower noise for images of *flaring* solar structures. A semi-empirical solution which allows to decrease the role of background noise in such cases is presented below.

4 Adjusting the Background

As discussed in Paper I, in order to avoid problems with negative signals after dark current subtraction, we introduced the dark current handling deep into the root of the ANDRIL algorithm. Appropriate correction has been inserted into the expression where the primary correction factors are evaluated (cf. Eq. (12), in Paper I). During the tests of the method, we noticed that by applying appropriate small additive ,,pedestal background" (B) it is possible to lessen the influence of input noise on the deconvolved pattern of intensities. While applying pedestal we substitute Eq. (12), in Paper I with the following expression:

$$\varphi_{\rm new} = \varphi_{\rm old} \frac{\sum_{\rm i} P({\rm i}|{\rm j}) \frac{O({\rm i})}{C({\rm i})+D({\rm i})+B}}{\sum_{\rm i} P({\rm i}|{\rm j})} \ , \tag{2}$$

where B represents a constant arbitrary non-negative array. The other quantities in Eq. (2) are as follows (cf. Paper I): $\varphi_{\rm new}$ is the new approximation of the brightness distribution being calculated from the preceding distribution $\varphi_{\rm old}$, $P({\rm i}|{\rm j})$ is the PSF (the fraction of light coming from the true location "j" that gets scattered into the observed pixel "i"), O is the noisy input image, C is the corresponding calculated image and D is the dark current matrix.

The influence of pedestal B is noticeable for regions where $O < B$. Appropriate selection of pedestal value allows to effectively damp the statistical noise below the B level, as illustrated in Fig. 3. Provided $B \leq \sqrt{O}_{\rm max}$, we have found there is no noticiable (exceeding 5%) degradation of the deconvolution quality as characterized by the value of q parameter.

By introducing pedestal, as in Eq. (2), the count flux conservation property built into the maximum likelihood algorithm is however violated (without substantially challenging its convergence). In order to counteract this effect, an external total intensity renormalisation is necessary at the appropriate place in the code to provide overall conservation of the counts during deconvolution. The result of applying an extra pedestal is similar to using the technique of ,,thresholding" in order to increase the image contrast.

5 Error Propagation and Texture Effects

The results of performed tests indicate for presence (in case of over-resolved deconvolution) of so called ,,texture effects". These effects are connected to the ,,location" of the source relative to the input pixel grid. The texture pattern is the most noticeable in the rms fluctuations pictures presented in Fig. 2 (right column), however the effects are also present in the intensity patterns. It turns out that local maxima of emission in the deconvolved

Figure 3: Illustration of the effect of using the various levels of artificial pedestal while performing noisy image deconvolution. Upper left and right patterns represent the test intensity model and typical noisy input image respectively. The maximum level of the input signal $O_{\max}$ has been assumed to be ~ 100 (counts). The patterns below are the results of deconvolutions obtained from the same noisy input image but with increasing pedestal levels adopted (0, 5, 10 and 20 counts respectively).

patterns tend to be placed central relative to the input pixel grid. Relative intensity amplitudes due to texture effects are small, usually less than few percent. We have checked that relative scale of texture effects does not depend substantially on the assumed subsampling scheme. The texture is the most evident for regions with low input signal. We have found that texture effects can be practically neglected for regions where the deconvolved signal is >5.

By comparing, for each subpixel, the ratio of averaged deconvolved signal $\mathbf{S}$ and its rms amplitude $\mathbf{N}$, it is possible to investigate the distribution of „signal-to-noise" ($\mathbf{S/N}$) on the deconvolved brightness pattern. It is observed, as expected, that the relative uncertainties are the largest for those subpixels which are placed far from the maxima in the test pattern (regions of low input S/N).

Figure 4: Dependence of the deconvolved $\mathbf{S/N}$ on the value of $\mathbf{S}$ (upper panel) and on the value of input S/N (lower panel). See the text for details.

Based on numerous runs where the input signal has been a subject to noise we have determined the dependence of $\mathbf{S/N}$ (signal to rms noise after deconvolution) on the magnitude of deconvolved signal $\mathbf{S}$. The rms noise $\mathbf{N}$

has been determined (for all examples discussed below) based on 40 perturbed runs. In Fig. 4 (upper panel) we present log-log scatter plot of ($\mathbf{S/N}$) vs $\mathbf{S}$ containing 3×10^4 points representing collected results for three separate cases in which the maximum of intensity on the input image has been taken as 100, 1000 and 3000 (counts). It is seen that for deconvolved signal exceeding $1\div2$ (counts/subpixel) the Poisson character of the noise (after deconvolution) is preserved ($\mathbf{N}\sim\sqrt{\mathbf{S}}$).

This derived dependence allows to assign the rms uncertainty in the deconvolved image based entirely on the deconvolved value only i.e. one can assign uncertainty to each deconvolved intensity value. It is also seen from the dependence that below the threshold of few counts per subpixel, the results of deconvolution are rather uncertain.

It is instructive to correlate the ($\mathbf{S/N}$) after deconvolution with the (S/N) characteristic of the data (input image). By appropriate averaging of the ($\mathbf{S/N}$) over corresponding input pixel area (5×5 subpixels). The plot is shown in the bottom panel in Fig. 4. The horizontal axis represents in this case the (S/N) as taken from input while the vertical axis represents averaged deconvolved ($\mathbf{S/N}$). The 1200 points plotted represent the same three cases as in the panel above. Except for pixels placed at the edges of the deconvolution field (points lying off the main cloud), a well defined dependence is observed. This dependence indicates that for cases when the input $S/N\sim30$ (1000 counts/pixel) the corresponding value of $\mathbf{S/N}$ after deconvolution is $\sim$7 i.e. factor $4\div5$ times less. This large value of ,,noise enhancement factor" (NEF) is related partly to the ill-conditioned nature of the deconvolution and partly to pixel subsampling. After making appropriate exercises we have found that:

- deconvoluion with no subsampling corrects the brightness distribution substantially (the value of q parameter improves by more than an order of magnitude) and contributes the factor of ~2 to NEF,

- allowing for 3×3 subsampling causes NEF to increase again by similar factor of ~2. Further pixel fragmentation does not increase substantially $\mathbf{S/N}$ for adopted shape of PSF.

This behaviour of NEF illustrates the ill-conditioned nature of deconvolution problem.

6 Conclusions

The analysis performed indicates that:

- Presence of statistical noise substantially influences the results of deconvolution. For areas where the input signal to noise ratio S/N is low (below ~ 5) spurious isolated high contrast features tend to develop in the deconvolved image which limit the range of physical interpretation of the deconvolution results.

- For higher S/N (above 10) the results of deconvolution are however sufficiently stable against the input data noise which allows for realistic physical interpretation to the extent discussed in detail in Paper I.

- In order to properly estimate the uncertainty in each of intensity subelement after deconvolution it is feasible to run at least several "perturbation" exercises where the input data are statistically perturbed with the noise pattern changing randomly from one case to another.

- The uncertainties in deconvolved images acquire Poissonian character above certain threshold (1 count per subpixel in the case of assumed 5:1 pixel subsampling).

- Due to the ill-conditioned nature of deconvolution problem, the statistical noise present in the data propagates into the deconvolved image being relatively enhanced. For 1:1 deconvolution (no subsampling) the noise enhancement factor NEF is $\sim$2 for the SXT instrument. This value shoud be representative for any deconvolution technique used in order to remove the SXT instrument blurring.

- While allowing for subsampling one has to agree for another twofold increase of the noise propagation factor. In case of SXT instrument the overall NEF becomes then $4\div5$. Surprisingly, the value of the NEF does not grow for fainter selected subsampling.

- For areas in the image with low S/N values a development of texture pattern in the deconvolved image is observed. This texture is due to cyclically changing „distance" of particular subpixel from the input observation grid. The amplitude of texture does not depend substantially on the assumed subsampling. Following the described procedure it is possible to accommodate *a posteriori* at least for a part of texture effects.

- It is possible to efficiently limit the influence of data noise by averaging the input signal over several exposures (where appropriate).

- Results of empirical exercises indicate that by modifying the kernel of ANDRIL algorithm by introducing artificial additive component called „background pedestal" the noise reduction is quite effective for low intensity areas in the image of any single frame of interest.

Based on the results of present study one may estimate that in order to expect $\sim$20% accuracy in the deconvolved intensity image, one have to use the input data accurate to few percent. For SXT such a high signal accuracy is representative for regions with the *compressed* signal being above ~ 100 DN (data number, cf. Paper I) threshold. For most real SXT images, due to automatic exposure selection scheme, the brightest areas are usually well above this limit and therefore deconvolution offers new insight into detailed morphology of these bright sources.

Presented results of noise propagation within the ANDRIL algorithm help in better understanding of limits for the physical interpretation of deconvolution. It is possible now to assign uncertainty to each of the intensity values as determined using standard ANDRIL discussed in Paper I.

Image averaging can be used effectively in cases when the cadence of the observations is fast enough in order to assume the source brightness distribution unchanged between exposures. This is unfortunately usually not the case for SXT flare observing sequences. Therefore the „background pedestal" approach has been mostly used for massive SXT deconvolution task with value B of pedestal selected at approximately twice the level of the dark current.

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